15-213 Recitation 2

Introduction to Computer Systems

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Today

- Datalab
- Integers Review
- Floating Point Basics
- Examples

Datalab

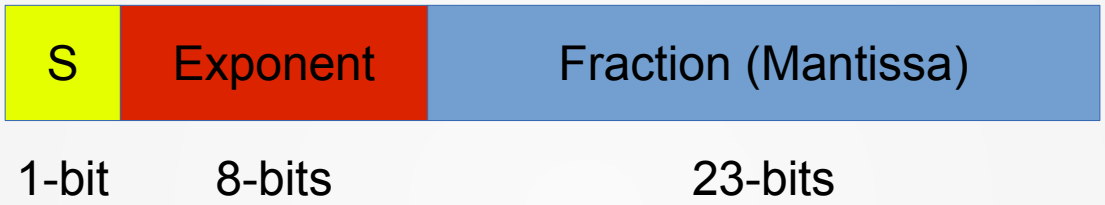
- Due: Monday, September 9 at 11:59 PM!
- Some interesting functions
  - Two's complement: ~x+1(Corner case)
  - Flipping a bit: XOR with 1
- Questions?

Integers

- Decimal ↔ Binary
- Signed vs Unsigned
- Two's complement
- Notes: Implicit Casting
  - Always cast to unsigned int incase you have mixed types in an expression
  - Is `-1 < 0`? What about `-1 < 0U`?
- Ranges

Floating Point

- Basic Format



$$-1^{S} \times M \times 2^{E}$$

Where E is based on the Bias

$$Bias = 2^{(k-1)} - 1 = 2^{(8-1)} - 1 = 127$$

k = exponent bits

Interpreting the Bits

- Exponent
  - 0000..000 → Denormalized Form
    - E = -Bias + 1
    - Frac = 0.FFFFF…
  - eeee..eee → Normalized Form
    - E = Exponent – Bias
    - Frac = 1.FFFFF….
  - 1111..111 → Special
    - If Frac = 0000.000 → Infinity
    - Else → NaN

Rounding

- Round to even
  - Like regular rounding except for the exactly half case
  - For exactly half case, check the LSB that will remain. If it is 1, round up, else round down

1.10	1001	Greater then 0.5, round up	1.11
1.10	0110	Less than 0.5, round down	1.10
1.11	1000	Round to even up	10.00
1.10	1000	Round to even down	1.10

Examples!

- We will use 8-bits for convenience

S EEEE FFF

8-bit floating point number

Examples!

- Convert: `5`

Examples! - Normalized Number to Float

- Convert: 5
  - S=0 (As 5 is positive)
  - Convert 5 to binary → $5_{10} = 101_2$
  - Normalized Value, so lets fit the leading 1

    $1.01_2 \times 2^2$
  - F = 010
  - E = 2 → Exp = E + Bias = 2 + 7 = $9_{10} = 1001_2$
  - Answer → 0 1001 101
  - What about -5?

Examples!

- Convert: 6/512

Examples! - Denormalized Number to Float

- Convert: 6/512
  - S=0
  - Convert to binary → $0.01171875_{10} = 0.00000011_2$
  - Denormalized → E = -Bias + 1 = -6

$$0.11 \times 10^{-6}$$

  - F = 110
  - Answer = 0 0000 110

Examples!

- A Simple trick:
  - Remember the fractional representation?

2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}	2^{-9}
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$	$\frac{1}{512}$

  - Lets write it in terms of 512

2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}	2^{-9}
$\frac{256}{512}$	$\frac{128}{512}$	$\frac{64}{512}$	$\frac{32}{512}$	$\frac{16}{512}$	$\frac{8}{512}$	$\frac{4}{512}$	$\frac{2}{512}$	$\frac{1}{512}$

  - We need 6 512th's, hence the binary is 000000110

Examples!

- Convert 27

Examples! - Rounding

- Convert 27
  - S = 0
  - Convert 27 to binary → 11011_2
  - Normalized, so → $1.1011_2 \times 2^4$
  - E = 4 → Exp = E + Bias = 4 + 7 = 11 = 1011_2
  - Frac → 1011 (Too many digits, we need to round)
    - 1011 → Round to even up → 110
  - Answer → 0 1011 110

Examples!

- Convert: 1 1001 010

Examples! - Normalized Float to Number

- Convert: 1 1001 010
  - Sign bit is 1, so negative
  - Exp = 1001, Normalized → E = Exp – Bias
    - $1001_2 = 9_{10} \rightarrow E = 9 - 7 = 2$
  - Frac → 010, hence Number is 1.010
  - $1.010 \times 2^2 \rightarrow 101_2 \rightarrow 5_{10}$
  - Answer → -5

Examples!

- Convert: 0 0000 110

Examples! - Denormalized Float to Number

- Convert: 0 0000 110
  - Sign bit says its positive
  - Denormalized, so E → -6
  - Frac → 110 → 0.110_2
  - Number → 0.110_2 x 2^{-6} → 0.00000011_2
  - Answer → 6/512

Examples!

- Convert: $0\ 1011\ 110_2$

Examples! - Rounding error

- Convert: $0\ 1011\ 110_2$
  - Sign bit says answer is positive
  - Normalized → Exp = $1011_2 = 11_{10}$ →
    - E = Exp – Bias = 11 – 7 = 4
  - Frac = 110
  - Number = $1.110 \times 2^4 \rightarrow 11100$
  - Answer → 11100_2 → 28!

Questions?

- New Office Hours:
  - Sunday : 1:00 PM → 2:30 PM
  - Thursday : 2:00 PM → 3:30 PM